Lesson 2. Interest Rates

1 Last time...

- A model for interest rates:
  - A_n = amount in our savings account at year n
  - r = annual interest rate
  - We assumed that interest is compounded annually
    - At the end of each year, we earn interest at the rate r on the amount we have in the account
  - DS:
$$A_{n+1} = A_n(1+r) \quad n = 0, 1, 2, \ldots$$
  - Solution:
$$A_n = A_0(1+r)^n \quad n = 0, 1, 2, \ldots$$

2 Compounding monthly

- What if we compound monthly instead of annually?
- The annual interest rate is still r
- At the end of each ______, we earn interest at the rate ______ on the amount we have in the account
- Model:

Example 1. Let's compare compounding annually vs. compounding monthly. Suppose our initial deposit is \$100, and the annual interest rate is 0.05.

a. How much will be in our savings account after 10 years when interest is compounded annually?
b. How much will be in our savings account after 10 years when interest is compounded monthly?

3 Compounding more generally

- What if interest compounded weekly? Daily? Hourly?
- Suppose we split the year into k equal time periods
  - e.g. $k = 12$ for monthly, $k = 52$ for weekly
- So, t years = ______ time periods
- At the end of each ______, we earn interest at the rate ______ on the amount we have in the account
- Model:
- We can rewrite the solution to the DS in terms of years instead of time periods:

Example 2. Suppose our initial deposit is \$100, and the annual interest rate is 0.05. How much will be in our savings account after 10 years when interest is compounded weekly? Daily? Hourly?

	k	amount
annually		
monthly		
weekly		
daily		
hourly		

4 Compounding continuously

- When we split the year into k equal compounding time periods, the amount in our savings account after t years is

$$A_{kt} = A_0 \left(1 + \frac{r}{k}\right)^{kt}$$

- What happens when we make the time periods smaller and smaller, or as ?
- In general, as the number of compounding time periods in a year approaches infinity, we have
- This is the formula for continuously compounding interest

Example 3. Suppose our initial deposit is \$100, and the annual interest rate is 0.05. If our savings account continuously compounds interest, how much will be in the account after 10 years?

5 More examples

Example 4. Suppose the annual interest rate is 0.02, compounded daily. How much should we deposit initially so that we have \$10,000 in 20 years?

Example 5. Suppose we initially deposit \$5,000. What is the smallest annual rate which will let us accumulate at least \$6,000 total over 5 years if interest is compounded monthly?